Yiddish word of the day

uncle	grandma	דער פֿעטער	די באָבע
aunt	grandpa	די מומע	דער זיידע
cousin	sister	דער קוזין	די שוועסטער
sibling	son	די געשוויסטער	דער זון

=

Yiddish phrase of the day

"mit shnei ken men nisht makhn gulmolkhes" = מיט שניי קען מען נישט מאַכן גומאָלקעס!

"with snow one can't make cheesecake" =

TAKE THE EVALS!!!

Currently at 75%!!!

Orthogonal Stuff

First in $\mathbb{R}^n$

- How to use LA to talk about "distance" and "length"?

Def: Let $\vec{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$, $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ in $\mathbb{R}^n$.

$\sqrt{x^2+y^2}$

Define $\vec{u} \cdot \vec{v} := u_1v_1 + u_2v_2 + u_3v_3 + \dots + u_nv_n$

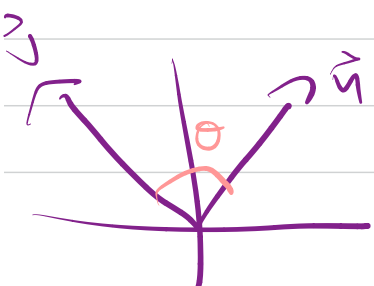
$\sqrt{u \cdot u}$

the dot product

Def: 1) Let $\vec{u}$ in $\mathbb{R}^n$. Then define the length of $\vec{u}$ to be $\sqrt{\vec{u}\cdot\vec{u}} = \|\vec{u}\|$

2) Let $\vec{u}, \vec{v}$ in $\mathbb{R}^n$. The distance between $\vec{u}, \vec{v}$ is defined to be $\|\vec{u}-\vec{v}\|$

- **Fact**: We have $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$ where θ is the angle between $\vec{u}, \vec{v}$.



"inner product spaces"
"Hilbert space"

Now let V be VS.

Can define "distance" between arbitrary vectors

Let $\mathcal{B}$ be basis. Then taking coordinate vectors gives vectors in $\mathbb{R}^n$

Def: Let v, w in V. Then the distance between v, w

$$\mathrm{dist}(v,w) := \| [v]_{\mathcal{B}} - [w]_{\mathcal{B}} \|$$

- We can therefore also talk about angles between arbitrary vectors.

Def 2: Let $\vec{u}, \vec{v}$ in $\mathbb{R}^n$. Then we say $\vec{u}, \vec{v}$ are orthogonal if $\vec{u} \cdot \vec{v} = 0$

- Say $(\vec{u}_1, \ldots, \vec{u}_r)$ are orthogonal if $\vec{u}_i \cdot \vec{u}_j = 0$

- **Fact:** If $(\vec{u}_1, \ldots, \vec{u}_r)$ are orthogonal then they are LI

Recall: If $\vec{b}$ in span$(\vec{v}_1, \ldots, \vec{v}_n)$ then

$$\vec{b} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$$

- However, to find these coefficients it's kind annoying.

- Unless the vectors $(\vec{v}_1, \ldots, \vec{v}_n)$ are orthogonal

In this case

$$\boxed{c_i = \frac{\vec{b} \cdot \vec{v}_i}{\vec{v}_i \cdot \vec{v}_i}}$$

Having a denominator is always annoying. Hence the following def

Def: An orthogonal sequence $(\vec{v}_1, \ldots, \vec{v}_n)$ is said to be orthonormal if $\vec{v}_i \cdot \vec{v}_i = 1$ for all i

Goal: Have an orthonormal basis for $\mathbb{R}^n$

Why?

- **Def**: We say $Q = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_n \\ \downarrow & & \downarrow \end{pmatrix}_{n \times n}$

  is an orthogonal matrix

  if the columns are orthogonal

Facts: Let $Q = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_n \\ \downarrow & & \downarrow \end{pmatrix}_{n \times n}$. Then the following are equivalent

1) Q is a orthogonal matrix

2) For any $\vec{v}$ in $\mathbb{R}^n$ $\|Q\vec{v}\| = \|\vec{v}\|$

3) For any $\vec{v}, \vec{w}$ in $\mathbb{R}^n$ $Q\vec{v} \cdot Q\vec{w} = \vec{v} \cdot \vec{w}$

What this means: Let Q be an orthogonal matrix

- Q preserves distance between vectors (by 2)
- Q preserves angles between vectors (by 3)

(Google "conformal maps" and "isometries")

Practically

- If you want to change a vector without changing it (move it around but not deform it) then an Orthogonal matrix is the way to go.

Facts

- Can always make a basis into ON basis

  (Grahm-Schmidt procedure)

- Let $A = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_n \\ \downarrow & & \downarrow \end{pmatrix}_{m \times n}$ with $\vec{v}_1 \ldots \vec{v}_n$ LI, Then can decompose A as

$$A = QR$$

with Q orthogonal

R "invertible "upper triangular" (in EF)

("QR factorization") (8.6 has applications of this)

(Sections 8.4 / 8.5 in book are really cool (especially 8.5) but we won't discuss them. Look at them though!!! Cool stuff!!!!!!)